Addendum: Finite Size Effects for Some Bootstrap Percolation Models[1]

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In a recent paper we presented a description of the approach to the infinite-system limit for some bootstrap percolation (BP) models. Our results for
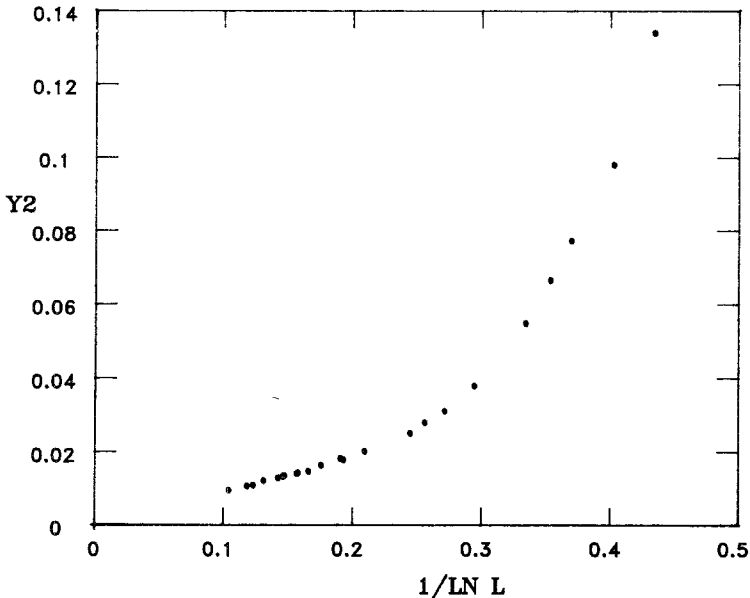

Fig. 1. A graph of $Y2 = (q_{50}^L)/[\ln(q_{50}^L)]^2$ as a function of $1/\ln L$ for $m = 2$ directed BP.

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one case, the $m = 2$ directed BP model, were inconclusive. For this model[1] it was observed in our paper that the numerical data appeared to give a straight line in agreement with scaling of the finite system threshold, p_{50}^L, toward the infinite-system limit p_c, according to the rule $1 - p_{50}^L \sim 1/(\ln L)^{1/2}$, whereas the improved exact result of Schonmann[2] should give $1/\ln L$ scaling with possible logarithmic corrections. We did not obtain a graph of the behavior with explicit logarithmic corrections, and we were unable to see the approach to the asymptotic $p_c = 1.0$.

A careful interpretation of Eq. (10) of our paper shows that a possible realization of the logarithmic corrections would give finite-size scaling of the form

$$\frac{q_{50}^L}{(\ln q_{50}^L)^2} \sim \frac{1}{\ln L} \tag{1}$$

where $q = 1 - p$ and $q_{50}^L = 1 - p_{50}^L$. A graph of this behavior is shown in Fig. 1, were we see that the system threshold approaches 1.0 as the system size becomes infinite. The approach is in a straight line for the larger samples. This supports the conjecture that logarithmic corrections are indeed present and makes the numerical results compatible with Schonmann's exact value $p_c = 1.0$.

ACKNOWLEDGMENTS

REFERENCES

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2. R. H. Schonmann, private communication.